

**Primordial black holes in braneworld cosmologies: Accretion after formation**

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We recently studied the formation and evaporation of primordial black holes in a simple braneworld cosmology, namely the Randall-Sundrum type II cosmology. Here we study the effect of accretion from the cosmological background onto the black holes after formation. While it is generally believed that in the standard cosmology such accretion is of negligible importance, we find that during the high-energy regime of braneworld cosmology accretion can be the dominant effect and lead to a mass increase of potentially orders of magnitude. However, unfortunately the growth is exponentially sensitive to the accretion efficiency, which cannot be determined accurately. Since accretion becomes unimportant once the high-energy regime is over, it does not affect any constraints expressed at the time of black hole evaporation, but it can change the interpretation of those constraints in terms of early Universe formation rates.

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**I. INTRODUCTION**

Primordial black holes (PBHs) are relics which may form in the early Universe [1] and survive to have effects in later epochs, thus shedding light on the physical processes operating during the Universe's early stages. In a recent paper (Guedens, Clancy and Liddle [2]), we considered the physics of black holes in a popular variant to the standard cosmology, the Randall-Sundrum type II braneworld cosmology [3]. We showed that the presence of the fifth (AdS) dimension could significantly modify the properties of black holes, and indeed that black holes evaporating at key epochs of the Universe might behave as higher-dimensional objects.

The results of that paper assumed that after formation the dominant process affecting the mass of the black holes was Hawking evaporation. In the standard cosmology it is believed that the neglect of accretion of material from the cosmological background is a very good approximation, except perhaps immediately after formation, though the calculations are rather uncertain. However, we remarked that it was not necessarily true that accretion would also be negligible during the high-energy regime. The purpose of this paper is to investigate accretion during the high-energy regime.<sup>1</sup> We will find that accretion can indeed be the dominant effect in the high-energy regime, though accurate calculation of its effects is presently impossible.

Our calculations follow the notation of Ref. [2], and we will consider both the standard cosmology and the Randall-Sundrum type II cosmology, which features a high-energy regime where the Friedmann equation is modified. The AdS radius of the extra dimension will be denoted  $l$ , and  $M_4$  and  $l_4$  indicate the usual four-dimensional Planck mass and

length (we set  $c=1$ ). The four-dimensional cosmological constant is set to zero. We use  $M$  for the black hole mass and  $r_0$  for the Schwarzschild radius. If  $r_0 < l$  then the black hole will be effectively five-dimensional and the appropriate five-dimensional Schwarzschild solution is used.

**II. ACCRETION FORMALISM**

Calculations of PBH accretion in the standard cosmology have a long history but are plagued with significant uncertainties. First of all, while it is known that black holes forming during radiation domination cannot be much smaller than the horizon size (otherwise pressure forces would prevent collapse), it is unclear how much smaller they might be. Secondly, it is unclear precisely how efficient a black hole might be at absorbing incoming radiation from a cosmological background, or how important the effect of backreaction might be. Results prove extremely sensitive to assumptions made concerning these quantities. While early work by Zel'dovich and Novikov [5] speculated that PBHs might even be able to grow as fast as the horizon, subsequent work, especially by Carr and Hawking [6,7], made a convincing case that such growth could not occur, and moreover that once the PBH became significantly smaller than the horizon accretion would become very inefficient. This view is now widely held, though there are papers where accretion is found to be important, including interesting work by Hacyan [8] in which incoming radiation was described by a Vaidya metric matched to a flat radiation-dominated Universe, and in which the mass was found to grow proportional to the horizon size [ $M(t) \rightarrow 0.06-0.08 t$ ] at late times even for very small initial mass under the idealization of perfect radial inflow.

There are different approaches to estimating the accretion rate. Clearly it will be proportional to the surface area of the black hole, and to the energy density of the radiation background; the key question is the constant of proportionality, to which our results will prove extremely sensitive. The rel-

<sup>1</sup>As we were completing this paper, a paper appeared by Majumdar [4], using the formalism of our earlier paper [2] to investigate this same question. Our results are in broad agreement, though we will highlight the key differences.

event length scales in the problem are the cosmological horizon size, the black hole radius, and potentially also the mean free path of the particles comprising the radiation background (which should be much less than the horizon size if the background is to be thermalized).

The two approaches available to estimate that constant are to use an absorption cross-section for radiation incident from a distance, or to consider the thermal properties of radiation near the event horizon. The former has the drawback that it assumes a long mean free path for the radiation (which cannot be true if the PBH radius is a significant fraction of the cosmological horizon), while in the latter the effect of the black hole geometry cannot readily be included. We will consider both and compare.

To use the cross section of the black hole, one needs to take into account that the impact parameter for absorption of incoming radiation by the black hole is greater than the event horizon size, as the black hole bends the particle trajectory towards it. The black hole therefore has an effective radius  $r_{\text{eff}}$  for capturing particles. In the standard cosmology  $r_{\text{eff}} = 3\sqrt{3} r_0/2$ , where  $r_0 = 2M/M_4^2$  is the Schwarzschild radius [9]. In the case of an effectively five-dimensional black hole, the equivalent expression is  $r_{\text{eff}} = 2r_0$  [10] where

$$r_0 = \sqrt{\frac{8}{3\pi}} \left( \frac{l}{l_4} \right)^{1/2} \left( \frac{M}{M_4} \right)^{1/2} l_4. \quad (1)$$

The accretion rate is estimated by assuming first of all that the radiation is non-interacting, so that the radiation reaching the black hole at time  $t$  has come from a known distance  $x$ . The fraction of the radiation starting at that point (assumed to have an isotropic momentum distribution) which is absorbed by the black hole is just the solid angle subtended by the black hole at that distance, and then adding up over all particles in a shell of width  $dt$  at that distance yields

$$\frac{dM}{dt} = \pi r_{\text{eff}}^2 \rho(t), \quad (2)$$

where the distance  $x$  drops out of the calculation. This expression will continue to hold even in the presence of interactions, at least as long as the particle mean free path is much larger than the black hole radius, because in thermal equilibrium particles are as likely to scatter onto trajectories towards the black hole as off those trajectories. However, it is unclear precisely what effect scattering near the black hole might give, particularly in regions where the black hole has significantly modified the geometry, and so we can expect this result to be modified by a factor of order unity. It is also unclear whether the depletion of the radiation in the vicinity of the black hole by the absorption might lead to a significant reduction in accretion efficiency. Finally, we note that this calculation is certainly naive in ignoring spin and frequency dependent effects in the absorption cross-section [11,12].

A little more insight can be obtained by considering the thermal balance between the black hole and the background if they were at the same temperature. The black hole radiation leads to a mass loss given by the Stefan-Boltzmann law applied to the radius  $r_{\text{eff}}$  [10], except that it is also known

that in the four-dimensional case the Stefan-Boltzmann law overestimates the emission by a factor 2.6 [11] due to so-called gray-body factors allowing for the finite size of the black hole as seen by long-wavelength radiation (the majority of the power is emitted at wavelengths comparable to the event horizon radius) and the inclusion of spin-dependent effects. Such gray-body factors would also apply to the absorption of long-wavelength radiation, and arguing that there should be balance when the cosmological and black hole temperature match suggests that in that situation Eq. (2) overestimates the absorption by a factor of around 2.6. However, the regime of interest for accretion is when the background temperature is much larger than the black hole temperature, and then the gray-body factors should be much less important, supporting the normalization of Eq. (2). Were the cosmological temperature smaller the gray-body factors would be of increasing significance, but in that limit evaporation dominates in any case.

Care is needed, however, in applying thermal balance arguments in the braneworld case, because while accretion is taking place on the brane, the black hole can evaporate into both brane and bulk; in effect the black hole provides a route for leakage of energy from the brane via evaporation of gravitons. Even if the black hole is in thermal balance on the brane, it will not be in equilibrium with the bulk, and so would still lose mass. However, the thermal balance argument can be used between the accretion and the evaporation onto the brane alone.

An alternative view is to consider the radiation background at a given time to extend smoothly all the way to the event horizon with an isotropic momentum distribution, and compute the flux entering the black hole. Consider a small shell of radius  $dt$  at the event horizon, which will contain an energy density  $4\pi r_0^2 \rho(t) dt$ . However, not all this radiation will be inwardly directed; the flux entering the black hole is one-quarter of that energy density yielding

$$\frac{dM}{dt} = \pi r_0^2 \rho(t). \quad (3)$$

This is a factor of a few smaller than the estimate above. However, this estimate is arguably more dubious. The approximation of an isotropic radiation distribution near the event horizon is unlikely to be good, as flux coming from the direction of the black hole will be absent, though one could argue that that this missing flux is correctly allowing for depletion of the radiation already absorbed by the black hole. More seriously, this estimate does not allow for the effect of the black hole geometry on the radiation, whereas the effective radius for absorption did in the previous calculation.

Summarizing the above, we can take the accretion rate to be

$$\frac{dM}{dt} = F \pi r_{\text{eff}}^2 \rho(t), \quad (4)$$

where  $F$  is a numerical constant measuring the accretion efficiency. It is conceivable that it might be as large as unity (particularly if the radiation mean free path is believed to be

much larger than the event horizon radius), whereas other arguments suggest it may be somewhat smaller. However, neither calculation is likely to be accurate if the black hole is a significant fraction of the horizon size, because then accretion is constrained by the amount of material actually accessible to the black hole. Unfortunately, it turns out that the results in the high-energy regime are exponentially sensitive to the value of  $F$ , and therefore we keep it in the calculations that follow. By contrast, the recent paper by Majumdar [4] assumed  $F=1$  throughout.

### III. PRIMORDIAL BLACK HOLE ACCRETION

#### A. Accretion in the standard cosmology

As a warm up, we can apply the above formalism in the standard cosmology where, however, it will turn out that the approximations made are unlikely to be valid. Suppose a PBH is formed at a time  $t_i$  in the early phases of a radiation-dominated Universe, with mass

$$M_i = f M_H(t_i) = f M_4^2 t_i. \quad (5)$$

Here  $M_H(t_i)$  is the horizon mass and  $f < 1$  denotes what fraction of the horizon mass the black hole comprises. Using Eq. (4) with the four-dimensional Schwarzschild solution and the radiation energy density

$$\rho = \frac{3M_4^2}{32\pi t^2} \quad (6)$$

gives

$$\frac{dM}{dt} = \frac{81}{32} F M_4^{-2} \frac{M^2}{t^2}. \quad (7)$$

Integrating from a time  $t_i$  when the black hole mass is  $M_i$  leads to

$$M(t) = M_i \left[ \frac{t/t_i}{A t/t_i + (1-A)} \right], \quad A \equiv 1 - \frac{81}{32} F f. \quad (8)$$

The behavior of  $M(t)$  crucially depends on the sign and magnitude of the factor  $A$ . If it is positive, the black hole mass asymptotes to

$$M_\infty = \frac{M_i}{A}. \quad (9)$$

If  $F$  and/or  $f$  are small (small efficiency or initial PBH size), the factor will be close to 1, and the asymptotic mass will not be much bigger than the initial mass. But if  $A$  is small ( $Ff \rightarrow 0.4$ ), the black hole mass will grow nearly as fast as the horizon mass, until  $t/t_i \approx 1/A$  or until the radiation-dominated regime comes to an end. The factor  $A$  can even be negative for  $Ff > 0.4$ , in which case Eq. (8) gives a diverging mass in a finite time, violating causality and thus certainly indicating that the approximations used have broken down.

However, in order to obtain a final mass that is much bigger than the initial mass, it is required that the initial PBH

radius must be close to 74% ( $=0.4^{1/3}$ ) of the horizon radius (for efficiency  $F=1$ ), or more (for  $F<1$ ). For such large PBHs use of the quasi-static approximation (where the black hole is represented as a series of Schwarzschild solutions as the mass increases), becomes questionable. But in the case of PBHs the initial size of the black hole *must* be of the order of the horizon, to overcome the fluid pressure. Carr and Hawking [6] proved there was no black hole solution embedded in a Friedmann background in which the black hole could grow at the same rate as the cosmic horizon. They then concluded that the black hole could only grow less fast than the horizon. After a short period of time its size *would* have become much smaller than the horizon, at which point the quasi-static approximation could be used to confirm no further accretion. This result has become widely accepted, though see Refs. [8,13].

#### B. Accretion in the high-energy regime of the braneworld scenario

We now turn to the high-energy regime of the braneworld scenario. If black holes are to behave as five-dimensional ones, we showed [2] that they must form during the high-energy regime, and that it was possible for such PBHs to survive even to the present. In order to interpret possible observational signatures, we therefore need to understand accretion in the high-energy regime. In this section we consider only accretion, and in the following section we will consider the combined effects of accretion and evaporation.

If a black hole forms at a time  $t_i$  in the high-energy regime of the Randall-Sundrum type II scenario, its size will necessarily be smaller than the AdS radius  $l$ , and hence described as a five-dimensional Schwarzschild black hole. We take its initial mass to be a fraction  $f$  of the horizon mass (now computed using formulas relevant to the high-energy regime [2])

$$M_i = f M_H(t_i) = 16f M_4 \left( \frac{l}{l_4} \right)^{-1} \left( \frac{t_i}{t_4} \right)^2. \quad (10)$$

The growth rate is again given by Eq. (4), where we must now employ the energy density in the high-energy phase

$$\rho = \frac{3}{32\pi} \frac{M_4^2}{t_c t}, \quad (11)$$

with the transition time between high-energy and standard regimes given by  $t_c = l/2$ , and use the five-dimensional expressions for the event horizon radius and capture crosssection. This gives

$$\frac{dM}{dt} = \frac{2F}{\pi} \frac{M}{t}. \quad (12)$$

Integrating from the time of formation onwards results in

$$M(t) = M_i \left( \frac{t}{t_i} \right)^{2F/\pi}. \quad (13)$$

In this case we see that the black hole mass always grows less fast than the horizon mass  $M_H(t) \propto t^2$  (provided  $F$  is not significantly greater than unity), in contrast to the situation in the standard cosmology. Accordingly, the approximations made in the calculation become more reliable as time goes by. Our result agrees with that of Majumdar [4] who, however, assumed  $F = 1$  throughout.

This calculation shows that, regardless of the initial mass, black holes can experience significant growth, with the power-law depending sensitively on the accretion efficiency  $F$ . The uncertainty in its precise value lends considerable uncertainty to the accretion growth of the PBHs during the high-energy phase.

Once the high-energy phase ends, the cosmological background evolution becomes the standard one, but the PBH remains in the five-dimensional regime. To determine whether accretion still continues, we now need to apply Eq. (4) with the five-dimensional PBH properties with the standard cosmological evolution, taking as the initial condition the mass of the PBH at the transition time  $t_c$  and bearing in mind that the black hole will be much smaller than the horizon size at this point. Using these equations gives

$$M(t) = M(t_c) \exp\left[\frac{2F}{\pi} \left(1 - \frac{t_c}{t}\right)\right], \quad (14)$$

indicating a further growth by only a factor of order unity (comparable to the expected uncertainties in the calculation).

We therefore conclude that accretion swiftly becomes unimportant as the standard cosmology sets in for five-dimensional black holes as well as four-dimensional ones. Accordingly, it is the slower decrease of the background density during the high-energy regime which makes accretion important, rather than the change in black hole properties.

#### IV. COMBINING ACCRETION AND EVAPORATION

We now take both accretion and evaporation into account in the high-energy regime<sup>2</sup>

$$\frac{dM}{dt} = \left(\frac{dM}{dt}\right)_{\text{acc}} + \left(\frac{dM}{dt}\right)_{\text{evap}}, \quad (15)$$

where

$$\left(\frac{dM}{dt}\right)_{\text{acc}} = \frac{q}{2} \frac{M}{t} \quad (16)$$

and

$$\left(\frac{dM}{dt}\right)_{\text{evap}} = -\frac{\tilde{g} M_5^3}{2M} \quad (17)$$

(see Ref. [2]), where for convenience we have defined

<sup>2</sup>Some work loosely related to this, but concerning astrophysical accretion by collisionally produced mini black holes in a TeV gravity model, can be found in Ref. [14].

$$q \equiv \frac{4F}{\pi}. \quad (18)$$

Assuming the four-dimensional cosmological constant vanishes, the relation between the five-dimensional fundamental mass scale and the Planck mass is given by

$$M_5 = M_4 \left(\frac{l}{l_4}\right)^{-1/3}. \quad (19)$$

In the above expression we have defined

$$\tilde{g} \approx \frac{0.0062}{G_{\text{brane}}} g_{\text{brane}} + \frac{0.0031}{G_{\text{bulk}}} g_{\text{bulk}} \quad (20)$$

where  $g_{\text{brane}}$  is the usual number of degrees of freedom into which the black hole can evaporate, while  $g_{\text{bulk}} = \mathcal{O}(1)$  is the number of bulk degrees of freedom (in the simplest case just the five polarization states of the graviton). For the most part, the black hole's energy is lost through Hawking radiation on the brane. As an example we mention the case where the black hole emits only massless particles, for which  $g_{\text{brane}} = 7.25$  and  $\tilde{g} = 0.023$ . Unlike in Ref. [2], we have written the gray-body factors  $G_{\text{brane}}$  and  $G_{\text{bulk}}$  explicitly; in the standard cosmology the gray-body factor is equal to 2.6, but precise values are not known for the braneworld. For thermal balance between the accretion and the evaporation onto the brane, the accretion efficiency  $F$  should equal  $1/G_{\text{brane}}$  when the temperatures are equal; however, as discussed in Sec. II the absorption gray-body factors should approach one if the background temperature is much greater than the black hole temperature.

In the standard cosmology Eq. (15) does not have an analytical solution, but it does in the present case:

$$M(t)/M_i = \left\{ \left(\frac{t}{t_i}\right)^q - \frac{\tilde{g}}{4\sqrt{f}} \frac{1}{1-q} \frac{M_5^{3/2}}{M_i^{3/2}} \left[ \left(\frac{t}{t_i}\right) - \left(\frac{t}{t_i}\right)^q \right] \right\}^{1/2}. \quad (21)$$

An expression of this form was found by Majumdar [4], though again for the specific case of efficiency  $F = 1$  (i.e.  $q = 4/\pi$ ). However, a very slight change in the efficiency can change the qualitative behavior. If  $q > 1$  (efficiency  $F > 0.78$ ) the PBH will steadily grow until the cosmological transition time  $t_c$  is reached (provided  $M_i > M_5$ ). If  $q < 1$  (efficiency  $F < 0.78$ ), the mass loss term will grow faster than the mass gain term. However, assuming that the initial black hole mass is large compared to the fundamental mass scale,  $M_i \gg M_5$ , the loss term will initially be much smaller than the gain term. In other words, if a black hole forms with a mass of order the horizon mass at a time  $t_i \gg t_5$ , then the initial black hole temperature will be much lower than the temperature of the radiation background and evaporation can initially be neglected.

For many choices of parameters, evaporation is then negligible throughout the high-energy regime. But for sufficiently low accretion efficiency, the mass growth can come



to a halt whilst the black hole is still in the high-energy regime.<sup>3</sup> This ‘halt’ time is obtained from  $dM/dt=0$  and reads

$$\left(\frac{t_h}{t_i}\right)^{1-q} = q \left[ 1 + (1-q) \frac{4\sqrt{f}}{\tilde{g}} \left(\frac{M_i}{M_5}\right)^{3/2} \right]. \quad (22)$$

If  $M_i \gg M_5$  we can neglect the first term, to obtain

$$\left(\frac{t_h}{t_i}\right)^{1-q} \approx q(1-q) \frac{4\sqrt{f}}{\tilde{g}} \left(\frac{M_i}{M_5}\right)^{3/2}. \quad (23)$$

In order for the derivation to be valid the halt time must satisfy  $t_h < t_c$ , or equivalently

$$\left(\frac{t_i}{t_4}\right)^{4-q} < \frac{\tilde{g}}{f^2} \frac{2^{q-9}}{q(1-q)} \left(\frac{l}{l_4}\right)^{2-q}. \quad (24)$$

The above condition will be satisfied if  $l$  is large enough (a long high-energy regime) and/or if  $M_i$  is small enough (a high initial PBH temperature), depending on the value of  $q$ . Put another way, for a given  $l$  and  $M_i$  there is a minimum accretion efficiency which ensures that neglecting the mass loss through evaporation is justified all the way up to  $t = t_c$ .

As an example we bear in mind that the AdS radius  $l$  is constrained by experiment as

$$l < 10^{31} l_4. \quad (25)$$

If there was a period of high-energy inflation, the requirement that gravitational waves do not lead to excessive anisotropies in the cosmic microwave background (CMB) leads to a lower limit on the horizon mass (and hence on the PBH mass) at the end of inflation, namely [2]

$$M_i > 2 \times 10^6 M_5. \quad (26)$$

For the extreme values of  $M_i$  and  $l$  (and hence for all other values), we find that an efficiency better than 30% ( $q > 0.39$ ) is sufficient to ensure mass growth all the way up to the cosmic transition time.

<sup>3</sup>In the limit  $q \rightarrow 1$  the mass growth decreases logarithmically, and if  $M_i \gg M_5$ , evaporation can be neglected until the standard regime is reached.

We now ask what is the total lifetime  $t_{\text{evap}}$  of a black hole that reaches a halt time in the high-energy regime. Provided we can trust Eq. (21) until total evaporation  $M(t_{\text{evap}}) = 0$ , we find

$$t_{\text{evap}} = q^{1/(q-1)} t_h. \quad (27)$$

Unless  $q$  is very close to zero, we see that the total lifetime will be of the same order as the halt time (taking into account the possibility that the final stage of the black hole’s lifetime is in the standard regime only *decreases* the estimate for  $t_{\text{evap}}$ , as it turns off the accretion term). Such black holes evaporate long before any observational constraints can be brought to bear, and we therefore conclude that PBHs in the regime where accretion in the high-energy regime can halt are not of interest.

To summarize, for a PBH with a lifetime  $t_{\text{evap}} \gg t_c$  we need consider only two situations in analyzing their evolution forwards in time. For black holes forming after the end of the high-energy regime, accretion is never important. For those forming within the high-energy regime, it is always a good approximation to neglect evaporation up until the cosmic transition time, after which accretion can be ignored and evaporation dominates.

## V. CONCLUSIONS

We have explored the possibility that PBHs might accrete from the cosmological background in the case of braneworld cosmology, and found, in agreement with a recent paper by Majumdar [4], that significant growth is possible. However, we have highlighted the extreme sensitivity of the resulting growth to the assumed accretion efficiency, which cannot be accurately computed. Accretion therefore adds considerable uncertainty to the evolution of individual PBHs after formation in the braneworld scenario.

Since accretion ends once the standard cosmology is restored (whether the PBHs are effectively four-dimensional or five-dimensional), we stress that accretion does not have any implications for interpreting observations in terms of the PBH density *at evaporation*. It does, however, impact on how those constraints are interpreted in terms of formation rates in the early Universe. We will be providing a detailed analysis of observational constraints on braneworld PBHs in a forthcoming paper [15].

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